[3]Bokar, J., Silverberg, L., and Özisik, M. N., "An Engineering Foundation for Controlling Heat Transfer in One-Dimensional Transient Heat Conduction Problems," *International Communications in Heat and Mass Transfer*, Vol. 22, No. 6, 1995, pp. 849–858.

[4]Silverberg, L., "Uniform Damping Control of Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, 1986, pp. 221–227.

Disturbance Rejection Approach to Actuator and Sensor Placement

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I. Introduction

FOR various reasons as discussed elsewhere,[1] the selection of actuator and sensor positions is still ad hoc. This is especially true for flexible structures, for which many candidate configurations can exist. This study is an attempt to make the selection process more methodical.

One approach to actuator and sensor placement is to optimize a closed-loop performance metric directly by selecting the actuators, sensors, and controller gains simultaneously. This direct approach makes sense if the desired closed-loop performance is well defined. Because the individual actuator and sensor contributions to the closed-loop performance metric are complex, the solution strategy usually employs nonlinear programming with many design and numerical iterations. A second approach is to select actuators and/or sensors on the basis of open-loop properties so that closed-loop performance is indirectly optimized. Because the individual sensor and actuator contributions to the open-loop metric are simple, nonlinear optimization usually is not needed. This approach will suggest efficient actuator and sensor configurations for any type of control law. The suggested method falls into the latter class of approaches.

The use of a Hankel singular value (HSV) formula as an actuator placement metric[2] is extended to flexible structures in discrete time.[3] A main novelty introduced is that the ambiguity in the weighting of the principal modes is addressed by incorporating the general disturbance rejection goal into the actuator and sensor placement formulation. Optimal actuator and sensor placement is considered for the purpose of designing control laws for the general disturbance rejection problem. This apparent restriction to the disturbance rejection problem is not too restrictive because it is well known from modern multivariable control theory that stability and even robustness requirements can be transformed to this form with appropriate weighting. Simulation results demonstrate that the improvement in closed-loop performance is independent of the type of controller used because open-loop properties have been improved.

II. Actuators and Sensors for Disturbance Rejection

Figure 1 shows a schematic of the disturbance rejection problem where the inputs to the plant consist of two vectors and, similarly, the outputs consist of two vectors. As a necessary starting point in a disturbance rejection problem, the disturbance source (or inputs) and the output response defining the performance to be optimized usually are defined by the requirements of the control problem.

The disturbance rejection viewpoint taken in this study is similar to the mode selection viewpoint proposed earlier in the context of model reduction for flexible structures.[4] The preceding formulation is also consistent with the general multivariable control design,[5] in which the plant $P = [P_{11}, P_{12}; P_{21}, P_{22}]$ is assumed to be given and the problem is to design a stabilizing and realizable controller, K, to minimize a suitable norm of the closed-loop transfer function matrix, $F_l(P, K)$. The basic difficulty in the actuator and sensor placement problem is that only P_{11} is given.

HSVs are used to construct a metric that quantifies the degree of controllability and observability for a given set of sensor and actuator configurations. Although the use of HSV to analyze the degree of controllability and observability of a linear system is well established, especially in model reduction applications,[4,6,7] the approximate decomposition of the HSV with multiple sensors and actuators in terms of the HSV of all combinations of sensor-actuator pairs is new. This result significantly simplifies the design problem of selecting the most effective set of sensors and actuators for flexible structures. The main novelty of the placement strategy is that the HSVs from the disturbance to the performance outputs are used to weigh the HSVs between candidate actuator and sensor sets. This is possible because, in both cases, individual HSVs directly correspond to individual structural modes, which is unique to flexible structures.

III. Decomposition of HSV

Given the quadruple (A_z, B_z, C_z, D_z) of a discrete linear time-invariant state-space matrix of a flexible structure, assumed to be lightly damped with distinct eigenvalues, let $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ denote the 2×2 block diagonal form whose ith block is

$$\tilde{A}_i = \begin{bmatrix} \mathrm{Re}(z_i) & -\mathrm{Im}(z_i) \\ \mathrm{Im}(z_i) & \mathrm{Re}(z_i) \end{bmatrix} \tag{1}$$

where (z_i, v_i) denote the ith eigenvalue and eigenvector pair of A_z, and let T denote the sampling period. The steady-state discrete-time controllability gramian $W_{c\infty}$ and the observability gramian $W_{o\infty}$ satisfy the Sylvester equations. The triple $(\tilde{A}, \tilde{B}, \tilde{C})$ is internally balanced if its gramians are equal and diagonal,[7] i.e., $W_{c\infty} = W_{o\infty} = \Gamma^2$. The nonnegative real diagonal elements of Γ^2 are called the HSVs of the system.

Because of the diagonal dominance property of the discrete controllability and observability gramian for flexible structures, the square of the ith HSV[3] is given by

$$\gamma_i^4 \cong \frac{\mathrm{tr}[\tilde{B}\tilde{B}^T]_{ii}\, \mathrm{tr}[\tilde{C}^T\tilde{C}]_{ii}}{(4\delta_i T)^2} \tag{2}$$

where $\delta_i = -(1/T)\mathrm{Re}(\ell n z_i)$ and the subscript ii denotes the ith 2×2 block of matrices formed from the inputs and outputs. It is shown[3] that the approximate formula is quite accurate up to frequencies near 90% of the Nyquist frequency.

For p actuators and q sensors, the input and output matrices consist of p columns and q rows, respectively:

$$B_z = \left[B_{z_1}, \ldots, B_{z_p}\right]; \qquad C_z^T = \left[C_{z_1}^T, \ldots, C_{z_q}^T\right] \tag{3}$$

Partition the state transformation matrix as follows:

$$V = [r_1, \ldots, r_n]; \qquad V^{-T} = \left[l_1^T, \ldots, l_n^T\right] \tag{4}$$

where $r_i = [\mathrm{Re}(v_i), -\mathrm{Im}(v_i)]$ and l_i^T are $2n \times 2$ matrices made up of components of ith right and left eigenvectors. The approximate

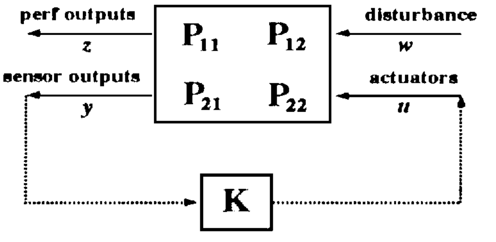

Fig. 1 General disturbance rejection problem.

Received March 4, 1996; revision received May 3, 1996; accepted for publication June 24, 1996.

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square of the ith HSV in Eq. (2) can be written in terms of individual actuator and sensor

$$\gamma_i^4 \cong \frac{\sum_{j=1}^{p} f_{ij}^2 \cdot \sum_{k=1}^{q} g_{ik}^2}{(4\delta_i T)^2} = \sum_{j=1}^{p}\sum_{k=1}^{q} \gamma_i^4(j,k) \tag{5}$$

where

$$f_{ik}^2 = \left\| l_i B_{z_k} \right\|_2^2; \qquad g_{ik}^2 = \left\| C_{z_k} r_i \right\|_2^2 \tag{6}$$

$\gamma_i^4(j,k)$ denotes the squared ith HSV for the jth input and kth output.

The importance of Eq. (5) lies in the decomposition of the HSV of the multivariable flexible structure in terms of the sum of approximate HSV of each actuator and sensor pair. The contribution of each pair appears in a very convenient form from their placement standpoint. The exact HSVs are not decomposable in general, i.e.,

$$\gamma_i^4 = \lambda_i \left(\sum_{j=1}^{p}\sum_{k=1}^{q} W_{cj} W_{ok} \right) \neq \sum_{j=1}^{p}\sum_{k=1}^{q} \lambda_i (W_{cj} W_{ok}) \tag{7}$$

IV. Placement Metric

Let Γ_{wz}^2 denote the HSV from the disturbance to the performance outputs (see Fig. 1):

$$\Gamma_{wz} = \mathrm{diag}\big(\gamma_{wz_1}, \ldots, \gamma_{wz_n}\big) \tag{8}$$

Also let Γ_{uy}^2 and $\bar{\Gamma}_{uy}^2$ denote the HSV for p actuators to q sensors and the HSV from a reference set of actuators to a reference set of sensors, respectively:

$$\Gamma_{uy} = \mathrm{diag}\big(\gamma_{uy_1}, \ldots, \gamma_{uy_n}\big) \tag{9}$$

$$\bar{\Gamma}_{uy} = \mathrm{diag}\big(\bar{\gamma}_{uy_1}, \ldots, \bar{\gamma}_{uy_n}\big) \tag{10}$$

Define the actuator and sensor placement metric as a weighted sum

$$J \triangleq \sum_{i=1}^{n} \frac{\gamma_{uy_i}^4}{\bar{\gamma}_{uy_i}^4} \gamma_{wz_i}^4 \cong \sum_{j=1}^{q}\sum_{k=1}^{p} J_{jk} \tag{11}$$

where contribution from jth sensor and kth actuator pair is

$$J_{jk} = \sum_{i=1}^{n} \left(\frac{\gamma_{wz_i}^4}{\bar{\gamma}_{uy_i}^4} \right) \gamma_{uy_i}^4(j,k) \tag{12}$$

It is necessary to introduce the above weights because the basic idea is to improve the joint controllability and observability for multiple modes, and not all modes are equally important physically. HSV can vary by orders of magnitude and, therefore, a few physically irrelevant modes could dominate an unweighted scalar metric. Hence, the normalizing weight $\bar{\gamma}_{uy_i}^4$ is introduced to make each HSV equally important, assuming no a priori physical knowledge of individual modes. An example of a normalizing weight is when all actuators and sensors are included. After the normalization, the relative importance of each mode in the control problem can be incorporated by using the weight, $\gamma_{wz_i}^4$. Modes that are important in the disturbance rejection performance will give larger weights. Note that J_{jk} is basically a weighted trace of $\Gamma_{uy}^4(j,k)$.

For actuator (sensor) placement only, the set of q sensors (p actuators) is assumed to be fixed. The above metric simplifies to

$$J \cong \sum_{k=1}^{p} J_k^{\mathrm{act}} \left(\sum_{j=1}^{q} J_j^{\mathrm{sen}} \right) \tag{13}$$

where the contribution of the kth actuator (jth sensor) is

$$J_k^{\mathrm{act}} = \sum_{i=1}^{n} f_{ik}^2 w_i^2 \left(J_j^{\mathrm{sen}} = \sum_{i=1}^{n} g_{ij}^2 w_i^2 \right) \tag{14}$$

Table 1 Actuator and sensor selection table

	Actuator number					Sensor placement
Sensor number	J_{11}	$\cdots$	J_{1k}	$\cdots$	J_{1p}	J_1^{sen}
	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	J_{j1}	$\cdots$	J_{jk}	$\cdots$	J_{jp}	J_j^{sen}
	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	J_{q1}	$\cdots$	J_{qk}	$\cdots$	J_{qp}	J_q^{sen}
Actuator placement	J_1^{act}	$\cdots$	J_k^{act}	$\cdots$	J_p^{act}	$J = \sum_{j=1}^{q}\sum_{k=1}^{p} J_{jk}$

The weighting factor for the ith mode is

$$w_i^2 = \frac{1}{(4\delta_i T)^2} \frac{\gamma_{wz_i}^4}{\bar{\gamma}_{uy_i}^4} \sum_{j=1}^{q} g_{ij}^2 \left[\frac{1}{(4\delta_i T)^2} \frac{\gamma_{wz_i}^4}{\bar{\gamma}_{uy_i}^4} \sum_{k=1}^{p} f_{ik}^2 \right] \tag{15}$$

Note that f_{ik} (g_{ij}) denotes the influence of the kth actuator (jth sensor) on the ith mode, and other factors that contribute to the above weighting factor include damping, significance with respect to disturbance rejection, and the reference HSV, for the ith mode.

V. Placement Strategy

Table 1 shows the individual contributions for all candidate sensors and actuators. The sum of each row is given by the last column, whereas the sum of each column is given by the last row. The total metric for all sensors and actuators is given by the sum of the last row or column. The costs J_k^{act} (J_j^{sen}) represent the contribution of the kth actuator (jth sensor) using all given sensors (actuators). The goal is to maximize the metric J using the least number of actuators and/or sensors. Note that the advantage of applying the metric to flexible structures is that the contribution of each actuator and sensor appears linearly and independently.

For simultaneous actuator and sensor selection problems, we need to maximize the linear sum of nonnegative numbers

$$J = \sum_{j=1}^{q}\sum_{k=1}^{p} J_{jk} \tag{16}$$

Clearly, a table of nonnegative numbers needs to be examined such that J can be approached using the fewest number of actuators and sensors. For an actuator-only (or sensor-only) selection problem, a bar chart of

$$J = \sum_{k=1}^{p} J_k^{\mathrm{act}} = \sum_{j=1}^{q} J_j^{\mathrm{sen}} \tag{17}$$

will indicate the best selection. Actuators (sensors) with small values of $J_k^{\mathrm{act}}(J_j^{\mathrm{sen}})$ can be removed as the least significant ones.

VI. Concluding Remarks

The metric and methodology introduced in this Note appears promising in its potential development as a generic tool for actuator and sensor selection for feedback-control design of flexible structures. This optimism is based on the advantages evident in using the approximate HSV formula for actuator/sensor selection: HSVs of individual pairs can be summed; and it is analytically and computationally simple and avoids direct numerical optimization; and, perhaps most importantly, it gives physical insight. This method also gives approximately optimal selection with respect to any number of sensors and actuators.

Preliminary simulation results based on H_2 and H_∞ control-law designs indicate that closed-loop performance can be improved significantly and the performance improvement is independent of the particular type of control law. This consistent improvement is clearly the result of judiciously improved controllability and observability in the open-loop system. However, the particular form specified for the disturbance rejection performance requirements strongly affects optimal actuator selection.

References

[1]Lim, K. B., and Gawronski, W., "Actuator and Sensor Placement for Control of Flexible Structures," *Control and Dynamics Systems*, edited by C. T. Leondes, Vol. 57, Academic, San Diego, CA, 1993, pp. 109–152.

[2]Gawronski, W., and Lim, K. B., "Balanced Actuator and Sensor Placement for Flexible Structures," *International Journal of Control* (to be published); also AIAA Paper 95-3259.

[3]Lim, K. B., and Gawronski, W., "Hankel Singular Values of Flexible Structures in Discrete Time," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 6, 1996, pp. 1370–1377; also AIAA Paper 96-3757, 1996.

[4]Gregory, C. Z., Jr., "Reduction of Large Flexible Spacecraft Models Using Internal Balancing Theory," *Journal of Guidance, Control, and Dynamics*, Vol. 7, No. 6, 1984, pp. 725–732.

[5]Boyd, S. P., and Barratt, C. H., *Linear Controller Design: Limits of Performance*, Prentice–Hall, Englewood Cliffs, NJ, 1991, Chap. 2.

[6]Gawronski, W., and Williams, T., "Model Reduction for Flexible Space Structures," *Journal of Guidance, Control, and Dynamics,* Vol. 14, No. 1, 1991, pp. 68–76.

[7]Moore, B. C., "Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction," *IEEE Transactions on Automatic Control*, Vol. 26, No. 1, 1981, pp. 17–32.

Slewing Multimode Flexible Spacecraft with Zero Derivative Robustness Constraints

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Introduction

THE problem of controlling flexible space structures in the presence of modeling uncertainties is an area of active research. For systems equipped with on-off actuators such as reaction jets, the problem is compounded because the command is discontinuous. Techniques for generating on-off command profiles have received much attention recently. One method, known as input shaping, is implemented by convolving a sequence of impulses, an input shaper, with a desired system command to produce a shaped input that is then used to drive the system.[1,2] If the impulse amplitudes are set to specific values and convolved with a step, then the resulting command profile will be a series of pulses realizable with on-off actuators.[3]

The impulse amplitudes and time locations are determined by satisfying a set of constraint equations. If the constraints require only zero residual vibration, then the resulting shaper is called a zero vibration (ZV) shaper. The earliest incarnation of ZV shaping was the technique of posicast control developed by Smith in the 1950s.[2] Singer and Seering developed robust input shaping by also setting the derivative with respect to the frequency of the residual vibration equal to zero.[1] The resulting shaper is called a zero vibration and derivative (ZVD) shaper.

Presented as Paper 96-3845 at the AIAA Guidance, Navigation, and Control Conference, San Diego, CA, July 29–31, 1996; received Aug. 22, 1996; revision received Sept. 28, 1996; accepted for publication Sept. 28, 1996.

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The idea of generating robustness by setting the derivative of the final state equal to zero was used by Banerjee[4] to slew large space-based antennae, by Liu and Wie[5] to produce on-off commands, and by Liu and Singh[6] to generate commands for nonlinear systems. Singhose et al.[3] proposed an alternative robustness technique.

This Note investigates on-off input shapers for multimode flexible spacecraft. Characteristics of the solution space for two-mode systems are discussed and then the robustness to modeling errors are quantified. We show that the robustness to errors in the second mode is highly dependent on the mode ratio.

Multimode On-Off ZVD Input Shaping

The impulses that compose an on-off input shaper are determined by satisfying five types of constraints: amplitude constraints, residual vibration constraints, robustness constraints, rigid-body constraints, and the requirement of time optimality.

A multiswitch bang-bang profile, which can be used with on-off actuators, can be generated by convolving a step input with an input shaper of the form[3]

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 & -2 & \cdots & -2 & 1 \\ 0 & t_2 & t_3 & t_4 & \cdots & t_{n-1} & t_n \end{bmatrix} \tag{1}$$

A_i and t_i are the amplitudes and time locations of the impulses and n is the number of impulses. Equation (1) sets the impulse amplitudes; the impulse times are determined by the remainder of the constraint equations.

The constraint on residual vibration amplitude can be conveniently expressed as the ratio of residual vibration amplitude with shaping to that resulting from a step input. For the kth mode of natural frequency, ω_k, and damping ratio, ζ_k, this percentage vibration is given by[1]

$$V(\omega_k, \zeta_k) = \exp(-\zeta_k \omega_k t_n)\sqrt{[C(\omega_k, \zeta_k)]^2 + [S(\omega_k, \zeta_k)]^2} \tag{2}$$

where

$$C(\omega_k, \zeta_k) = \sum_{i=1}^{n} A_i \exp(\zeta_k \omega_k t_i) \cos\left(\omega_k \sqrt{1-\zeta_k^2}\, t_i\right) \tag{2a}$$

$$S(\omega_k, \zeta_k) = \sum_{i=1}^{n} A_i \exp(\zeta_k \omega_k t_i) \sin\left(\omega_k \sqrt{1-\zeta_k^2}\, t_i\right) \tag{2b}$$

The zero vibration constraints are then k versions of Eq. (2) with V set equal to zero.

In addition to limiting residual vibration amplitude, ZVD shaping requires some amount of robustness to modeling errors by setting the derivative with respect to the frequency of the residual vibration equal to zero.[1] That is,

$$0 = \frac{\mathrm{d}}{\mathrm{d}\omega_k}[V(\omega_k, \zeta_k)] \tag{3}$$

Constraints on the rigid-body motion are also needed. For a system modeled as a series of masses, springs, and dampers, the rigid-body velocity is

$$v_d = \int_0^{t_n} \frac{u(t)}{M}\,\mathrm{d}t \tag{4}$$

where v_d is the desired terminal velocity, $u(t)$ is the input force, and M is the total system mass. For rest-to-rest slewing, v_d equals zero at the end of the slew, $t = t_n$. Integrating Eq. (4) gives a constraint on move distance, x_d:

$$x_d = \int_0^{t_n}\int_0^{t_n} \frac{u(t)}{M}\,\mathrm{d}t\,\mathrm{d}t \tag{5}$$

In a flexible rotary system, the transient deflection may cause a time-varying moment of inertia. These cases may require a more general form of rigid-body constraints.

Because of the transcendental nature of Eqs. (2) and (3), there will be multiple solutions. To make the solution time optimal, the shaper